Error minimization in Variable Fractional delay FIR Digital Filter

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Abstract:-The Digital filters having changeable frequency responses are called variable digital filters. Basically, variable digital filters include those with variable magnitude response or/and VFD response. The various efficient methods have also been developed for designing both FIR VFD filters and IIR all-pass VFD filters. The simplest VFD filters are the Lagrange-type VFD filters that can be derived from the Lagrange-polynomial interpolation. Lagrange-type VFD filters are a special class of FIR VFD filters whose frequency responses at frequency are the maximally flat, so they are often called the maximally flat VFD filters. Since Lagrange-type VFD filter has a closed-form impulse response that can be expressed as a polynomial in the VFD parameter, it is easy to use. Moreover, coefficient symmetry can be developed and exploited in fast hardware implementation through coefficient transformations. However, one disadvantage of the Lagrange-type VFD filters is that the passband width is rather narrow. For wideband VFD filtering applications, efficient methods for designing wideband VFD filters using both FIR and IIR all-pass transfer functions have been developed. So far, most existing design methods obtain VFD filters by approximating the desired (ideal) VFR in the WLS sense or minimax sense. The former minimizes the total error energy (integral of squared errors) of the VFR while the latter minimizes the maximum absolute error (peak error) of the VFR. Generally speaking, there is a trade-off between the two kinds of errors, i.e., the two kinds of errors cannot be simultaneously minimized by using the existing design methods. Usually, minimizing one then increases another. In the minimax design case, So develop a biminimax method for designing even-order FIR VFD filters whose VFR peak error and VFD peak error are simultaneously suppressed. More concretely, both the two peak errors are simultaneously made nearly equi-ripple, which is referred to as bi-equiripple. The most important part of the biminimax design is to linearize the highly non-linear constraints on the VFD errors as linear ones. After the linearization, the biminimax design can be easily performed by minimizing a mixed error function that contains both the VFR peak error and VFD peak error. However, minimization is a highly nonlinear problem, So we want to try iterative method for overcome it to further reduce the maximum absolute group-delay error in the least squares design, an iterative weighting-updated technique is also proposed, which constitutes the **outer** loop of the overall iterative process while the iteration stated earlier makes up the inner loop.

Keyword: - Finite impulse response(FIR), Infinite-impulse-response(IIR),Variable frequency response (VFR), Weighted-least-squares(WLS),Variable fractional delay(VFD).

1 INTRODUCTION

THE variable fractional delay (VFD) digital filters as an important class of the variable digital filters have been receiving increasingly attention in the past decade. Under tuning a controlling parameter, this kind of filters changes continuously a delay, which is a fraction of the sampling period.VFD filters have many applications in different areas of signal processing and communication, for example, time adjustment in digital receivers, speech coding and synthesis, time delay estimation and analog–digital (A/D) conversion, etc. A method for developing VFD filters is also an essential technique for the fractional linear discrete-time systems. Theoretically speaking, the design of variable digital filters under optimal sense is more complicated and difficult than the design of fixed delay filters, since the impulse response or the poles and zeros of the filters are some type of functions in the variable parameter (are generally assumed to be polynomial functions. Therefore, suboptimal approaches for the design of variable digital filters should be investigated for the purpose of reducing the computation complexity. For instance, the two-stage approach, i.e., designing a set of fixed-coefficient filters, and then fitting each of the coefficients as polynomials, has been proposed in the literatures. Recently, advances have been made on the design of some type of VFD filters, such as finite-impulse response (FIR) VFD filters and infinite-impulse response (IIR) all pass VFD filters. However, most of the methods employed iteration algorithm to formulate the design problem. Since large numbers of coefficients should be designed, related iteration algorithms still feature considerable computation complexity.

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2 DESIGN METHOD

For the purpose of comparison and the use of the conventional LS design of VFD FIR digital filters is reviewed in this section. The desired response of a VFD FIR filter is given by

$$H_d(w,p) = e^{-j(I+p)w} = e^{-jIw}(\cos(wp) - j\sin(wp)),$$

$$w \leq w_p; \qquad -0.5 \leq p \leq 0.5$$

Where is a prescribed mean group delay and is the parameter used to adjust the group delay of a filter online. The used transfer function is characterized by

$$H(z,p) = \sum_{n=0}^{N} h_n(p) z^{-n}$$

where coefficients $h(n)$ are expressed as the polynomials of p by

$$h_n(p) = \sum_{m=0}^{M} h(n,m)\, p^m$$

Hence

$$H(z,p) = \sum_{n=0}^{N}\sum_{m=0}^{M} h(n,m)\, p^m\, z^{-n}$$

$$= \sum_{m=0}^{M} G_m(z) p^m$$

Where sub filters $G_m(z)$ are represented by

$$G_m(z) = \sum_{n=0}^{N} h(n,m)\, z^{-n}, \qquad 0 \leq m \leq M$$

Obviously, this can be implemented by the Farrow structure .The equation can be further represented by

$$H_d(w,p) = e^{-jIw}\sum_{m=0}^{\infty}\frac{(-jpw)^m}{m!} \equiv \sum_{m=0}^{M}\left(\frac{(-jw)^m}{m!}e^{-jIw}\right)p^m$$

for sufficiently large M. After compersion, it can be found that the frequency response of $G_m(z)$ is used inherently to approximate $\left(\frac{(-jw)^m}{m!}e^{-jIw}\right)$ for $0 \leq m \leq M$. Therefore, it is reasonable to choose the coefficients of $G_m(z)$ to be symmetric for even and anti symmetric for odd M, and obviously, $I = N/2$. In this paper, only even is used, and the case for odd can be extended in a similar manner. Notice that the first subfilter $G_0(z)$ is designed to approximate $e^{-j(N/2)w}$, so $h(n,0) = \delta(n-(N/2))$. Hence, the frequency response of can be written as

$$H(e^{jw},p) = e^{-j\frac{N}{2}w}\Bigg[1 + \sum_{m=1}^{M_c}\sum_{n=0}^{N/2} a(n,m)p^{2m}\cos(nw) + j\sum_{m=1}^{M_s}\sum_{n=1}^{N/2} b(n,m)p^{2m-1}\sin(nw)\Bigg]$$

Defining

$$a = \left[\left[a(0,1)\,\ldots\ldots\ldots\ldots\ldots,a\left(\frac{N}{2},1\right)\ldots\ldots\ldots\ldots\ldots\; .a(0,M_c)\ldots\ldots\ldots.a\left(\frac{N}{2},M_c\right)\right]\right]^T$$

$$b = \left[\left[b(1,1)\,\ldots\ldots\ldots\ldots\ldots,b\left(\frac{N}{2},1\right)\ldots\ldots\ldots.b(1,M_s)\ldots\; \ldots\ldots.b\left(\frac{N}{2},M_s\right)\right]\right]^T$$

$$c(w,p) = \left[p^2\ldots\ldots\ldots p^2\cos\left(\frac{N}{2}w\right)\ldots\ldots\ldots\ldots.p^{2M_c}\ldots\ldots\; \ldots p^{2M_c}\cos\left(\frac{N}{2}w\right)\right]^T$$

$$s(w,p) = \left[p\sin(w)\ldots\ldots\; \ldots\ldots p\sin\left(\frac{N}{2}w\right)\ldots.p^{2M_{s-1}}\sin(w)\ldots\ldots\; .\ldots p^{2M_{s-1}}\sin\left(\frac{N}{2}w\right)\right]^T$$

Equation can be written as

$$H(e^{jw},p) = e^{-j\frac{N}{2}w}[1 + a^T c(w,p) + jb^T s(w,p)]$$

Where the subscript $.^T$ denotes the response operator.The conventional objective error function for designing a VFD FIR filter is given by

$$e_c(a,b) = \int_{-0.5}^{0.5}\int_{0}^{w_p} W(w)\,|H_d(w,p) - H(e^{jw},p)|^2$$

$$e_c(a,b) = \int_{-0.5}^{0.5}\int_0^{w_p} W(w)|\cos(pw) - j\sin(pw) - 1 - a^T c(w,p) - jb^T s(w,p)|^2 dwdp$$

$$e_c(a,b) = e_c(a) + e_c(b)$$

Where $W(w)$ is a weighting function

$$e_c(a) = \int_{-0.5}^{0.5}\int_0^{w_p} W(w)(\cos(pw) - 1 - a^T c(w,p))^2\, dwdp$$

$$e_c(a) = s_a + r_a^T a + a^T Q_a a$$

$$e_c(b) = \int_{-0.5}^{0.5}\int_0^{w_p} W(w)(\sin(pw) + b^T s(w,p))^2\, dwdp$$

$$e_c(b) = s_b + r_b^T b + b^T Q_b b$$

$$s_a = \int_{-0.5}^{0.5}\int_0^{w_p} W(w)(\cos(pw) - 1)^2 dwdp$$

$$r_a = -2\int_{-0.5}^{0.5}\int_0^{w_p} W(w)(\cos(pw) - 1)c(w,p)dwdp$$

$$Q_a = \int_{-0.5}^{0.5}\int_0^{w_p} W(w)c(w,p)c^T(w,p)dwdp$$

$$s_b = \int_{-0.5}^{0.5}\int_0^{w_p} W(w)(\sin(pw))^2 dwdp$$

$$r_b = 2\int_{-0.5}^{0.5}\int_0^{w_p} W(w)\sin(pw)\, s(w,p)dwdp$$

$$Q_b = \int_{-0.5}^{0.5}\int_0^{w_p} W(w)s(w,p)s^T(w,p)dwdp$$

For LS design $W(w) = 1$ and by applying the technique in [22], the elements inr_a,Q_a, r_b , and Q_b can be represented in closed form. In this paper K must be chosen large enough and $K = 10$ is used in this paper. Oncer_a , Q_a , r_b , and $Q_b$are obtained, the optimal solutions in the LS sense can be achieved by differentiating with respect to a and b , respectively, and then setting the results to zero as follows:

$$\frac{\partial e_c(a,b)}{\partial a} = \frac{\partial e_c(a)}{\partial a} = r_a + 2Q_a a = 0$$

$$\frac{\partial e_c(a,b)}{\partial b} = \frac{\partial e_c(b)}{\partial b} = r_b + 2Q_b b = 0$$

Which yield

$$a = -\frac{1}{2}Q_a^{-1} r_a$$

$$b = -\frac{1}{2}Q_b^{-1} r_b$$

3. PROPOSED DESIGN METHOD

In Section 2, the VFD FIR filter is designed such that the root-mean-square error of variable frequency response can be minimized. In this section, delay-oriented minimization is proposed so that the root-mean-square group-delay error can be minimized as much as possible while the desired variable frequency response can be preserved to a certain extent.

The desired group-delay response of a VFD FIR filter can be derived from

$$\tau_d(w,p) = -\frac{\partial}{\partial w} < H_d(w,p) = \frac{N}{2} + p \quad ; \quad |w| \le w_p : \ -0.5 \le p \le 0.5$$

And the actual group-delay response of the designed system is given by

$$\tau_H(w,p) = -\frac{\partial}{\partial w} < H(e^{jw},p) = \frac{\partial}{\partial w}\left(-\frac{N}{2}w + \tan^{-1}\frac{b^T s(w,p)}{1 + a^T c(w,p)}\right)$$

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$$\tau_H(w,p) = \frac{N}{2} - \frac{\big(1+a^T c(w,p)\big)\big(b^T s_d(w,p) - a^T c_d(w,p)\big)\big(b^T s(w,p)\big)}{\big(1+a^T c(w,p)\big)^2 + \big(b^T s(w,p)\big)^2}$$

$$c_d(w,p) = \frac{\partial}{\partial w} c(w,p)$$

$$s_d(w,p) = \frac{\partial}{\partial w} s(w,p)$$

The objective error function of the proposed method is given by

$$e(a,b) = \int_{-0.5}^{0.5}\int_0^{w_p} W(w)|\tau_d(w,p) - \tau_H(w,p)|^2\, dwdp + \alpha \int_{-0.5}^{0.5}\int_0^{w_p} W(w)|H_d(w,p) - H(e^{jw},p)|^2 dwdp$$

$$e(a,b) = e_\tau(a,b) + \alpha e_c(a,b)$$

Where α is a relative weighting constant, $e_c(a,b)$ and $e_\tau(a,b)$ is shown as

$$e_\tau(a,b) = \int_{-0.5}^{0.5}\int_0^{w_p} W(w)\left|p + \frac{\big(1+a^T c(w,p)\big)\big(b^T s_d(w,p) - a^T c_d(w,p)\big)\big(b^T s(w,p)\big)}{\big(1+a^T c(w,p)\big)^2 + \big(b^T s(w,p)\big)^2}\right|^2 dwdp$$

Obviously, minimization of is a highly nonlinear problem, and an iterative method is proposed in this paper to replace it.

The objective error function in the kth iteration for the proposed iterative method is represented by

$$e_k(a_k,b_k) = e_{\tau,k}(a_k,b_k) + \alpha e_{c,k}(a_k,b_k)$$

$$e_k(a_k,b_k) = \int_{-0.5}^{0.5}\int_0^{w_p} \frac{W(w)}{H_{k-1}^4(w,p)}\Big[\big(H_{k-1}^2(w,p)p + H_{R,K-1}(w,p) b_k^T s_d(w,p) - H_{I,k-1}(w,p) a_k^T c_d(w,p)\big)^2\Big] dwdp + \alpha(s_a + r_a^T a_k + a_k^T Q_a a_k + s_b + r_b^T b_k + b_k^T Q_a b_k)$$

Where the coefficient vectors denoted by subscript k are to be determined in the kth iteration and the functions denoted by subscript $k-1$ are the results of the previous iteration, which are defined by

$$H_{R,k-1}(w,p) = 1 + a_{k-1}^T c(w,p)$$

$$H_{I,k-1}(w,p) = b_{k-1}^T s(w,p)$$

$$H_{k-1}(w,p) = \big(H_{R,k-1}^2(w,p) + H_{I,k-1}^2(w,p)\big)^{\frac{1}{2}}$$

Thus, the original nonlinear problem can be converted into an iterative quadratic problem whose error function can be formulated into

$$e_k(a_k,b_k) = s_\tau + b_k^T Q_s b_k + a_k^T Q_c a_k + r_s^T b_k + r_c^T a_k + a_k^T Q_{CS} b_k + \alpha(s_a + r_a^T a_k + a_k^T Q_a a_k + s_b + r_b^T a_k + b_k^T Q_b b_k)$$

Where

$$s_\tau = \int_{-0.5}^{0.5}\int_0^{w_p} W(w) p^2 dwdp$$

$$Q_s = \int_{-0.5}^{0.5}\int_0^{w_p} W(w)\frac{H_{R,k-1}^2(w,p)}{H_{k-1}^4(w,p)} s_d(w,p) s_d^T(w,p) dwdp$$

$$Q_c = \int_{-0.5}^{0.5}\int_0^{w_p} W(w)\frac{H_{I,k-1}^2(w,p)}{H_{k-1}^4(w,p)} c_d(w,p) c_d^T(w,p) dwdp$$

$$r_s = 2\int_{-0.5}^{0.5}\int_0^{w_p} W(w)\frac{H_{R,k-1}(w,p)p}{H_{k-1}^2(w,p)} s_d(w,p) dwdp$$

$$r_c = -2\int_{-0.5}^{0.5}\int_0^{w_p} W(w)\frac{H_{I,k-1}(w,p)p}{H_{k-1}^2(w,p)} c_d(w,p) dwdp$$

$$Q_{CS} = -2\int_{-0.5}^{0.5}\int_0^{w_p} W(w)\frac{H_{R,k-1}(w,p)H_{I,k-1}(w,p)}{H_{k-1}^4(w,p)} . c_d(w,p) s_d^T(w,p) dwdp$$

In the kth iteration, solutions a_k and b_k can be obtained by differentiating with respect to a_k and b_k, respectively, and then setting the results to zero

$$\frac{\partial e_k(a_k,b_k)}{\partial a_k} = 2Q_c a_k + r_c + Q_{cs} b_k + \alpha r_a + 2\alpha Q_a a_k = 0$$

$$\frac{\partial e_k(a_k,b_k)}{\partial b_k} = 2Q_s b_k + r_s + Q_{cs}^T a_k + \alpha r_b + 2\alpha Q_b b_k = 0$$

Which lead to

$$a_k = -\frac{1}{2}(Q_c + \alpha Q_a)^{-1}(r_c + \alpha r_a + Q_{cs} b_k)$$

$$b_k = -\frac{1}{2}(Q_s + \alpha Q_b)^{-1}(r_s + \alpha r_b + Q_{cs}^T a_k)$$

After modification

$$a_k = \left[2Q_c - \frac{1}{2}Q_{CS}(Q_S + \alpha Q_b)^{-1}Q_{CS}^T + 2\alpha Q_a\right]^{-1}$$

$$\left[\frac{1}{2}Q_{CS}(Q_S + \alpha Q_b)^{-1}(r_s + \alpha r_b) - r_c - \alpha r_a\right]$$

$$b_k = \left[2Q_S - \frac{1}{2}Q_{CS}^T(Q_S + \alpha Q_a)^{-1}Q_{CS} + 2\alpha Q_b\right]^{-1}$$

$$\left[\frac{1}{2}Q_{CS}^T(Q_C + \alpha Q_a)^{-1}(r_c + \alpha r_a) - r_s - \alpha r_b\right]$$

Notice that because the related matrices, whose inverses are to be determined are symmetric and positive definite, the technique of Cholesky factorization can be applied to solve the ill-conditioning problem.

To terminate the iterative process, the relative norms are defined by

$$\beta_{a,k} = \frac{\|a_k - a_k - 1\|}{\|a_k\|}$$

$$\beta_{b,k} = \frac{\|b_k - b_k - 1\|}{\|b_k\|}$$

When both $\beta_{a,k}$ and $\beta_{b,k}$ are small enough, e.g., smaller than ε_{inn} , where ε_{inn} is a preassigned very small positive constant, the iterative process can stop.

The iterative procedures are shown in Fig. 1 and described in detail as follows.

Step 1) Given ,M ,w_p , and and setting iterative counter $k = 0$, find the initial coefficient vectors a_0 and b_0.

Step 2) Increase iterative counter k by one and calculate

$H_{k-1}(w,p)$,$H_{R,k-1}(w,p)$,$H_{I,k-1}(w,p)$,Q_S , Q_C, r_s ,r_c and Q_{CS}.

Step 3) Find coefficient vectors a_k and b_k.

Step 4) Check whether both relative norms $\beta_{a,k}$and$\beta_{b,k}$ are small enough by

$$\beta_{a,k} < \varepsilon_{inn}$$

$$\beta_{b,k} < \varepsilon_{inn}$$

If the condition is satisfied, stop the process; otherwise, go to Step 2).

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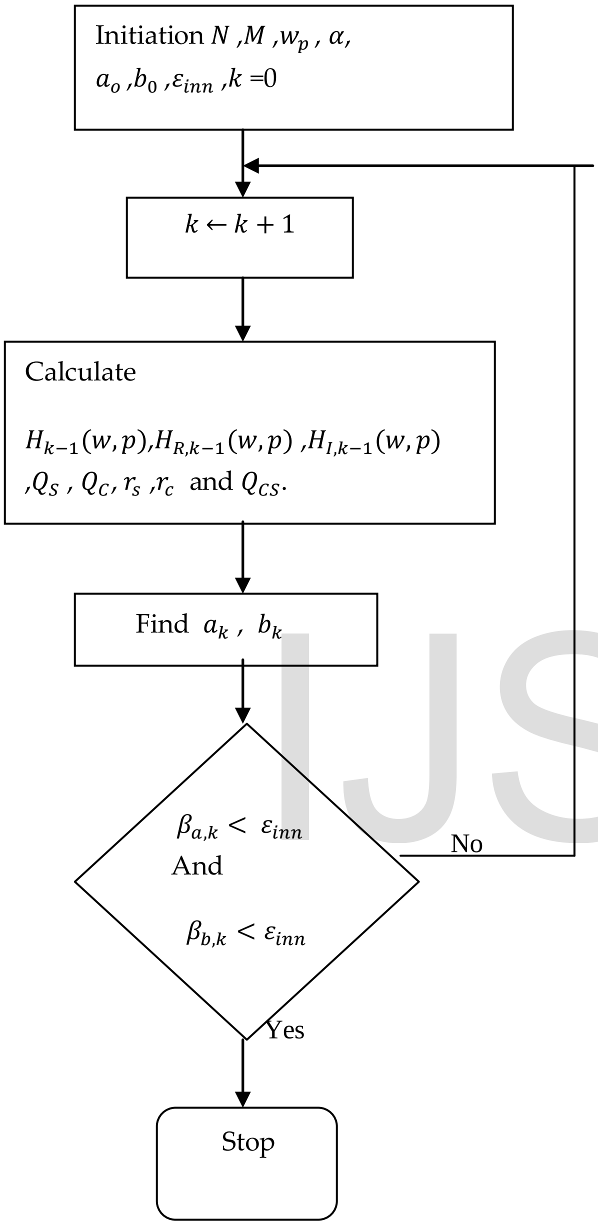

Flow Chart for proposed method

4. CONCLUSION

In this paper, a new method for the minimization of the root-mean-square error of variable group-delay response has been proposed for the design of VFD FIR digital filters. To overcome the nonlinear optimization for minimization, the proposed iterative method can be successfully used, and the experimental results may be show that the performance in group-delay response and the convergence of the iterative method are satisfactory.

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